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Extremal black brane attractors on the elliptic curve

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Abstract
Reconsidering the analysis of the moduli space of $N = 2$ eight-dimensional supergravity coupled to seven scalars, we propose a new scalar manifold factorization given by $SO(2,2)/(SO(2) \times SO(2)) \times SO(2,1)/SO(2) \times SO(1,1)$. This factorization is supported by the appearance of three solutions of Type IIA extremal black p-branes ($p = 0, 1, 2$) with $\mathrm{AdS}_{p+2} \times \mathrm{S}^{6-p}$ near-horizon geometries in eight dimensions. We analyze the corresponding attractor mechanism. In particular, we give an interplay between the scalar manifold factors and the extremal black p-brane charges. Then we show that the dilaton can be stabilized by the dyonic black 2-brane charges.

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1. Introduction

In previous years, four-dimensional extremal black hole attractors have received an increasing attention in the context of supergravity theories embedded in superstrings and M-theory compactified on Calabi–Yau manifolds [1–21]. In the near-horizon geometry limit, some of the supergravity scalar fields, obtained from geometric deformations, can take fixed values in terms of the black hole charges. Several studies of the $N = 2$ attractor mechanism in Type IIA superstring on Calabi–Yau 3-folds reveal that in such a limit only the complexified Kähler moduli can be stabilized by the Abelian black hole charges. The remaining scalar fields corresponding to the complex structure deformations, the values of the **R**–**R** C-fields on three cycles, the dilaton and the axion, remain free and can take arbitrary values.

However, the situation in six dimensions, which is obtained from Type IIA superstring on the K3 surface, is somewhat different. In this model, the attractor mechanism deals with the full geometric moduli space including both the complexified Kähler and the complex structure deformations [22–25]. These geometric parameters can be combined with the **NS–NS** B-field values, on non-trivial two cycles of the K3 surface, to form a quaternionic scalar manifold. Using a matrix formulation for such a quaternionic part, the geometric moduli of the K3 surface and the value of the **NS–NS** B-field are determined by the Abelian black hole charges. However, the dilaton, being identified with a non-compact orthogonal direction (SO(1, 1)), is attracted at the horizon of the extremal black F-string in terms of its charges (electric and magnetic).

More recently, a special effort has been devoted to discuss extremal black branes in higher dimensional supergravities [26]. This concerns the intersecting attractors involving extremal black holes and black strings in $D > 5$ dimensional supergravities. In particular, effective potentials and entropy functions have been computed in terms of U-duality black brane charge invariants.

The main purpose of this paper is to reanalyze the moduli space of eight-dimensional supergravity with $N = 2$ supersymmetry. We study the attractor mechanism in the framework of Type IIA superstring on the elliptic curve. In particular, we consider the extremal black p-branes. The corresponding near-horizon geometries are given by products of AdS_{p+2} with $(6-p)$-dimensional real spheres S^{6-p}. In this work, we propose a new realization for the scalar manifold of $N = 2$ supergravity in eight dimensions. This moduli space realization involves three factors which correspond to the appearance of three solutions of eight-dimensional extremal black p-branes with $p = 0, 1, 2$. Motivated by the compactification on the K3 surface, we discuss the attractors of such extremal black objects. In particular, we establish a correspondence between the scalar manifold factors and the extremal black objects charges. Then, a special interest is devoted to the dyonic black 2-brane attractor. The minimum of its effective potential gives the value of the dilaton in the $\mathrm{AdS}_4 \times \mathrm{S}^4$ near-horizon geometry limit.

The organization of this paper is as follows. In section 2, we develop a new factorization for the scalar manifold of $N = 2$ eight-dimensional supergravity involving three factors. The identification of each factor is based on the appearance of three different extremal black p-branes. In section 3, we discuss the attractor mechanism of such black objects. In particular, we point out the existence of a link between the extremal black brane charges and the $N = 2$ scalar manifold factors in eight dimensions. The last section is devoted to discussion of the results and open questions.

2. On the moduli space of $N = 2$ supergravity in eight dimensions

In this section, we reconsider the analysis of the scalar manifold of $N = 2$ supergravity in eight dimensions. The embedding of extremal black p-branes in Type IIA superstring theory on the elliptic curve provides a new factorization scheme for the seven scalars. We begin first by a review on the study of the scalar manifold. We will restrict ourself to the bosonic sector. The field content of the model consists of a graviton, 6-vector fields, three 2-form gauge fields, one self-dual 3-form gauge field and seven scalars. This spectrum can be obtained from the reduction of M-theory by T^3 with the $SL(3,R) \times SL(2,R)$ U-duality group [27]. The full spectrum is given by

$$\left(G_{\mu\nu}, B^I_{\mu\nu}, A^{I\alpha}_\nu, C_{\mu\nu\rho}, L^\Lambda_I, L^\alpha_i\right), \qquad \Lambda, I = 1,2,3, \quad \alpha, i = 1,2. \tag{2.1}$$

L^I_α is the coset representative of $\frac{SL(3,R)}{SO(3)}$, while L^α_i is the coset representative of $\frac{SL(2,R)}{SO(2)}$. In the M-theory picture, the scalar fields are the coordinates of the following homogeneous

Table 1. This table gives the correspondence between the scalar manifold factors and extremal black p-brane charges in Type IIA superstring on the K3 surface.

Coset space	Black brane	Gauge symmetry
$\frac{SO(4,20)}{SO(4)\times SO(20)}$	Black holes (black 2-branes)	$U(1)^{24}$
$SO(1,1)$	Black F-string	U(1)

space:

$$\frac{SL(3,R)}{SO(3)} \times \frac{SL(2,R)}{SO(2)}. \tag{2.2}$$

This scalar manifold represents the deformation of the metric and the value of the eleven-dimensional supergravity 3-form on T^3. Up to an overall scale factor corresponding to the size of T^3, $\frac{SL(3,R)}{SO(3)}$ determines the choice of the metric of T^3. $\frac{SL(2,R)}{SO(2)}$ is coordinated by the volume of T^3 and the value the 3-form gauge field takes on it.

Because of the strong coupling limit duality, the spectrum of the above eight-dimensional supergravity can also be obtained from the reduction of Type IIA superstring by T^2 (that is an $S^1 \times S^1$ fibration). This manifold has an obvious flat metric which depends on the size of the two circles. The moduli space of T^2 contains a real parameter describing its size and a complex parameter controlling its shape. The compactification of the massless bosonic ten-dimensional Type IIA superstring fields,

$$\textbf{NS–NS} : G_{MN}, B_{MN}, \phi \qquad \textbf{R–R} : A_M, C_{MNK} \quad M, N, K = 0, \ldots, \tag{2.3}$$

gives the same spectrum discussed above.

Motivated by the results obtained from the K3-attractors, and based on the T-duality groups in Type II superstrings, we propose here a new factorization for the scalar manifold of $N = 2$ supergravity in eight dimensions involving three factors. This factorization will be related to the appearance of three extremal black p-brane solutions with $p = 0, 1, 2$. To do so, let us first recall some ideas about the K3-attractors. The corresponding $N = 2$ supergravity consists of, among other, $(4 \times 20) + 1 = 81$ scalar fields, a $U(1)^{24}$ gauge symmetry and one self-dual antisymmetric B-field. The scalars span the following manifold product:

$$\frac{SO(4,20)}{SO(4)\times SO(20)} \times SO(1,1). \tag{2.4}$$

Here $\frac{SO(4,20)}{SO(4)\times SO(20)}$ describes the geometric moduli space admitting a hyper-Kähler structure. The extra factor $SO(1,1)$ stands for the dilaton, defining the string coupling constant g_s. It turns out that the two coset factors appearing in (2.4) correspond to two different extremal black objects in six dimensions required by the electric/magnetic duality. If we have an electrically charged p-brane, the magnetically charged dual object is a q-brane such that

$$p + q = 2. \tag{2.5}$$

There are essentially two six-dimensional extremal black p-brane solutions defined by $p = 0, 1$ with $\mathrm{AdS}_{p+2} \times \mathrm{S}^{4-p}$ near-horizon geometries. $p = 0$ corresponds to $\mathrm{AdS}_2 \times \mathrm{S}^4$ describing an electric-charged black hole, dual to the magnetic black 2-brane with $\mathrm{AdS}_4 \times \mathrm{S}^2$ near-horizon geometry. $p = 1$ is associated with a dyonic black F-string whose near-horizon limit is $\mathrm{AdS}_3 \times \mathrm{S}^3$. From the study of the attractor horizon geometries of extremal black p-branes ($p = 0, 1$) in Type IIA superstring on the $K3$ surface [22], we obtain the following connection given in table 1.

Motivated by these results, we expect to have a similar correspondence in the $N = 2$ eight-dimensional supergravity theory embedded in Type IIA superstring compactified on T^2.

In this way, the analog of equation (2.5) reads as

$$p + q = 4, \tag{2.6}$$

and can be solved in three different ways like

$p = 0$ $(q = 4)$	Black holes (dual black 4-branes)
$p = 1$ $(q = 3)$	Black strings (dual black 3-branes)
$p = 2$ $(q = 2)$	Dyonic black 2-branes

The extremal near-horizon geometries of these objects are given by the products of AdS_{p+2} with real spheres S^{6-p}. In eight dimensions, they are classified into three categories.

(1) $p = 0$ corresponds to an $\mathrm{AdS}_2 \times \mathrm{S}^6$ describing the near-horizon geometry of electric-charged black holes. Their dual magnetic are black 4-branes with $\mathrm{AdS}_6 \times \mathrm{S}^2$ near-horizon geometries. The objects carry charges associated with the gauge invariant field strengths $F^i = \mathrm{d}A^i (i = 1, \ldots, 6)$ of the $N = 2$ supergravity theory.

(2) $p = 1$ is associated with $\mathrm{AdS}_3 \times \mathrm{S}^5$ describing near-horizon geometry of extremal black strings. They carry electric charges associated with 3-form field strengths $H^i = \mathrm{d}B^i (i = 1, \ldots, 3)$. The electric charge is proportional to the integral of $\star H$ over S^5 that encloses the string. The magnetic dual horizon geometry reads as $\mathrm{AdS}_5 \times \mathrm{S}^3$ and describes black 3-branes. They are charged under the gauge invariant 3-form field strengths $H^i = \mathrm{d}B$.

(3) $p = 2$ corresponds to a dyonic black 2-brane with $\mathrm{AdS}_4 \times \mathrm{S}^4$ near-horizon geometry. This object carries both electric and magnetic charges associated with the single gauge invariant 4-form field strength $G = \mathrm{d}C$.

We shall see that this classification could be related to the moduli space of Type IIA superstring on Calabi–Yau spaces. In this compactification, there are three different contributions classified as follows.

- The dilaton defining the string coupling constant.
- The geometric deformations of the Calabi–Yau space including the antisymmetric B-field of the **NS–NS** sector. Depending on the Calabi–Yau spaces, these parameters involve complex structure deformations, complexified Kähler deformations or both.
- The scalar moduli described by the values of the **R**–**R** gauge fields wrapped on non-trivial cycles in the Calabi–Yau spaces.

According to this observation and based on the K3-attractor results mentioned earlier, the moduli space of Type IIA superstring compactified on T^2 should have a *priori* three factors. They may be related to the existence of three solutions of extremal black p-branes with $p = 0, 1, 2$. Under this hypothesis, the corresponding scalar manifold should take the form

$$M_1 \times M_2 \times M_3. \tag{2.7}$$

The main physical motivation of this factorization is to solve the attractor equation of black objects in eight dimensions separately. In particular, we would like to deal with individual black attractor equations like in the case of Type IIA superstring on K3, where the moduli space (2.4) involves only two factors. It has been shown in that case each factor corresponds to an individual black attractor solution [22]. Here, we expect to have a similar situation with three kinds of attractor equations corresponding to the three different black objects ($p = 0, 1, 2$) which appear in eight dimensions. Our factorization is obtained from breaking the $\mathrm{Spin}(3) \times \mathrm{Spin}(2)$ R-symmetry involved in the moduli space realization (2.2) given by

Salam *et al* which involves only two factors. However, in that realization one factor is associated with several black attractor equations. The breaking of R-symmetry to $U(1) \times U(1)$ that we point out provides a moduli space factorization into three subspaces where each one is associated with only one attractor equation. This representation will be useful for solving such equations separately without using intersecting near-horizon geometries of black objects [26].

The identification of each factor of (2.7) can be obtained by the help of the conjecture given in [25]. This conjecture can be re-formulated in the framework of the elliptic curve as follows:

(1) The black hole charges fix only the geometric deformations of the elliptic curve (the complex structure deformation, the Kähler deformation and the **NS**–**NS** B-field on T^2).

$$p = 0 \quad \text{Black hole charges} \rightarrow \text{The geometric parameters.}$$

(2) The moduli related to the values of the **R**–**R** gauge vectors on one cycle of the elliptic curve should be fixed by black string charges

$$p = 1 \quad \text{Black string charges} \rightarrow \text{The } \mathbf{R\text{–}R} \text{ stringy moduli.}$$

(3) The dilaton could be fixed as usual by the dyonic state, which in this case is a black 2-brane

$$p = 2 \quad \text{Dyonic black 2-brane charges} \rightarrow \text{the dilaton.}$$

Based on this conjecture, we will identify each M_i factor of (2.7) with a coset space. The T-duality role in Type IIA superstring on the elliptic curve requires that one factor should be identified with

$$M_2 = \frac{SO(2,2)}{SO(2) \times SO(2)},$$

determined by $2 \times 2 = 4$ parameters controlling the metric deformation and the value of the **NS**–**NS** B-field on the elliptic curve. The T-duality group of Type II superstrings is $SO(2,2)$ and that of M-theory is $SL(3)$. This result can be generalized to the compactification on T^d. In this case, the geometric scalar fields parametrize the coset space $\frac{SO(d,d)}{SO(d)\times SO(d)}$. This space corresponds to the choices of the metric with $\frac{d(d+1)}{2}$ degrees of freedom and the values of the antisymmetric B-field with $\frac{d(d-1)}{2}$ contributions.

The dilaton is related to the string coupling

$$g_s \sim \exp(\phi).$$

This is invariant under the shift $\phi \rightarrow \phi + \alpha$. At some points of the moduli space of T^2, α can be related to the complexified Kähler parameter. In fact, it is related to the Riemanian volume of T^2 and the volume provided by the B-field. These two parameters are related by the $SO(1,1)$ group. In this way, the dilaton in eight dimensions can be represented, as in six dimensions, by a non-compact circle given by

$$M_1 = SO(1,1).$$

The two remaining parameters specifying the Wilson line on T^2 can be combined into a complex field. It can be represented by the coset space

$$M_3 = \frac{SO(2,1)}{SO(2)}.$$

This factor will be related to the black string solutions. In fact, there are three black string charges. From Type IIA superstring on elliptic curves viewpoint, one charge corresponds to the fundamental string living in the NS–NS sector; while the other two are related to the R–R sector. These charges can be rotated by the $SO(1, 2)$ isometry group.

Finally, the factorization (2.7) reads as

$$\frac{SO(2,2)}{SO(2)\times SO(2)}\times\frac{SO(2,1)}{SO(2)}\times SO(1,1). \tag{2.8}$$

It is worth commenting on a possible interrelation between this Type superstring factorization and the M-theory one. The factor $\frac{SO(2,2)}{SO(2)\times SO(2)}$ can be parametrized by two complex scalar fields. One of them can be identified with the complexified volume of T^3 of the M-theory compactification given by the $\frac{SL(2,R)}{SO(2)}$. The other complex scalar come from the $\frac{SL(3,R)}{SO(3)}$ coset space. The $SO(1, 1)$ factor can also be deduced from the last coset space. The remaining two scalar fields of $\frac{SL(3,R)}{SO(3)}$ can be combined in the factor $\frac{SO(2,1)}{SO(2)}$.

Next we will analyze the extremal black p-brane attractors which correspond to this new realization of the scalar manifold of $N = 2$ supergravity in eight dimensions.

3. Attractor mechanism on the elliptic curve

Here we discuss the attractor mechanism of extremal black branes that appear in Type IIA superstring compactified on the elliptic curve. We start by briefly recalling the main results obtained in the context of higher dimensional Calabi–Yau backgrounds. Consider Type IIA superstring on a Calabi–Yau n-folds with SU(n) $(n > 0)$ holonomy group. In the low energy limit, it leads to supergravity models with only 2^{6-n} supercharges. The near-horizon geometries of the extremal black p-branes dealt with here are given by the product of AdS spaces and real spheres as follows:

$$\mathrm{AdS}_{p+2}\times \mathrm{S}^{8-2n-p}. \tag{3.1}$$

The numbers n and p satisfy the constraint

$$2\leqslant 8-2n-p. \tag{3.2}$$

For the compactification on the Calabi–Yau n-folds, the electric/magnetic duality relating electric black p-branes to q-dimensional magnetic ones reads as

$$p+q=6-2n. \tag{3.3}$$

In the case of the Calabi–Yau 3-folds, this equation can be solved by $p = q = 0$ describing a dyonic black hole with $\mathrm{AdS}_2\times \mathrm{S}^2$ near-horizon geometry. The corresponding compactification leads to a four-dimensional $N = 2$ supergravity with eight supercharges, coupled to a $U(1)^{h_{1,1}+1}$ Abelian symmetry[5]. There are also scalars belonging to the vector multiplets and hypermultiplets. The complexified Kähler moduli space is associated with the vevs of the vector multiplet scalars. The complex structure deformations, the **R–R** gauge field contributions, the axion and the dilaton belong to the hypermultiplets. The scalars of the hypermultiplets do not play any role in the study of four-dimensional black hole attractors and can be ignored. In Type IIA superstring on Calabi–Yau 3-folds, several studies concerning the attractor mechanism show that in the $\mathrm{AdS}_2\times \mathrm{S}^2$ near-horizon limit only the complexified Kähler moduli can be fixed by the Abelian black hole charges [2–16]. This can be obtained by minimizing the black hole effective potential [17]. This potential appears in the action of the $N = 2$ supergravity coupled to the Maxwell theory in which the Abelian gauge vectors come from the reduction of the 3-form on two cycles of the Calabi–Yau three folds. The hypermultiplet scalars remain free and take arbitrary values near the horizon limit of black

[5] $h_{1,1}$ is the dimension of the complexified Kähler moduli space in Type IIA superstring on Calabi–Yau 3-folds.

holes. It is worth pointing out that the complex structure parameters have been fixed in the context of Type IIB superstring on the Calabi–Yau 3-folds.

However, the situation in six dimensions is somewhat different. It is obtained by the compactification on the K3 surface which is a Calabi–Yau 2-folds. This manifold has a mixed geometric moduli space involving both the Kähler and the complex structure deformations. These scalars together with the values of the **NS–NS** B-field are fixed by the Abelian charges of the extremal black holes with $AdS_2 \times S^4$ near-horizon geometry. In the $AdS_3 \times S^3$ near-horizon limit, the dilaton has been fixed by the charges (electric and magnetic) of thr extremal black F-string [22].

So far we have recalled lower dimensional results; now we consider the case of $n = 1$ corresponding to the reduction on the elliptic curve. The analysis we follow here is quite similar to the case of $N = (1, 1)$ supergravity in six dimensions obtained from the compactification on the K3 surface. A close inspection reveals that the six and eight dimensions share some similarities. In both cases, they involve extremal black extended objects. This objects are absent in the higher dimensional Calabi–Yau compactification $n \geqslant 3$.[6] Moreover, the metric deformations and the values of the B-fields can be collected in one coset space. They are given by $\frac{SO(4,20)}{SO(4)\times SO(20)}$ for the K3 surface and $\frac{SO(2,2)}{SO(2)\times SO(2)}$ for T^2.

3.1. Scalar manifold factors/extremal black p-brane charges correspondence

Here we analyze the correspondence between the above moduli space factors and the p-brane charges. In eight-dimensional $N = 2$ supergravity, the total Abelian gauge group is

$$U(1)_{b2b} \times U(1)^3_{bs} \times U(1)^6_{bh}. \tag{3.4}$$

The $U(1)_{b2b}$ factor is the Abelian gauge symmetry associated with the single field strength of the 3-from, the **R**–**R** 3-field which can be decomposed into a self-dual and anti-self-dual sectors leading to the electric and magnetic charges of the dyonic black 2-branes. The electric and magnetic charges can be rotated by $SO(1, 1)$ isotropy symmetry, and therefore, it corresponds to the dilaton scalar manifold.

The Abelian factor $U(1)^3_{bs}$ corresponds to three gauge field strengths $H^i = \mathrm{d}B^i$ $(i = 1, 2, 3)$. One of them comes from the **NS–NS** sector associated with the F-string, while the two other are obtained from the reduction of the **R**–**R** C-field on the two one cycles of the elliptic curve. In this way, $U(1)^3_{bs}$ gauge symmetry can be factorized as

$$U(1)^3_{bs} = U(1) \times U(1)^2. \tag{3.5}$$

This separation of the charges is governed by a $SO(2, 1)$ isotropy symmetry. A simple inspection suggests that this sector can be associated with the coset space $\frac{SO(2,1)}{SO(2)}$.

The last factor $U(1)^6_{bh}$ is the Abelian gauge symmetry associated with the six field strength 2-forms $F^i (i = 1, \ldots, 6)$ of the eight-dimensional supergravity multiplet. These vector fields arise not only from the **R–R** sector, as in the case of higher dimensional Calabi–Yau compactification, but also from the **NS–NS** sector due to the fact that $b_1(T^2) \neq 0$. One might factorize the $U(1)^6_{bh}$ gauge symmetry as

$$U(1)^6_{bh} = U(1)^2 \times U(1)^2 \times U(1) \times U(1). \tag{3.6}$$

These Abelian gauge fields can be obtained from the **NS–NS** and **R**–**R** sectors. The $U(1)^2 \times U(1)^2$ gauge sub-groups are obtained from the **NS–NS** sector. This part has only $SO(2) \times SO(2)$ isotropy symmetry. There are another $U(1) \times U(1)$ gauge factor which come from the **R**–**R** sector. It is worth recalling that these gauge fields are the **R**–**R** 1-form and the

[6] This can be easily seen from the equation (3.2).

Table 2. This table describes the relation between scalar manifold factors and extremal black p-brane charges in eight dimensions.

Coset space	Black objects	Gauge symmetry
$\frac{SO(2,2)}{SO(2)\times SO(2)}$	Black holes (black 4-branes)	$U(1)^6_{bh}$
$\frac{SO(2,1)}{SO(2)}$	Black strings (black 3-branes)	$U(1)^3_{bs}$
$SO(1,1)$	Black 2-brane	$U(1)_{b2b}$

reduction of the 3-form on the elliptic curve. With the presence of these fields, we shall see that there is an enhancement of the isometry group. This can be supported by the M-theory uplifting scenario. From M-theory point of view, the six gauge fields can be classified into two categories. Three of them are obtained from the metric, while the remaining three one come from the 3-form. In fact, the eleven-dimensional metric gives two vectors belonging to the **NS–NS** sector and one vector living in the **R–R** sector. The corresponding Abelian gauge charges can be rotated by the $SO(2,1)$ isotropy symmetry. The bosonic field content of the reduction of the eleven-dimensional 3-form consists of a similar vector contributions which are also rotated by $SO(2,1)$. The full isometry group should be $SO(2,1)\times SO(2,1)$. It is a well-known fact that $SO(2,1)\times SO(2,1)$ is equivalently to $SO(2,2)$, as we want. In this way, the above six charges can be related to the variation of the metric and the B-field on the elliptic curve ($\frac{SO(2,2)}{SO(2)\times SO(2)}$). This can be understood from the equivalence

$$\frac{SO(2,2)}{SO(2)\times SO(2)}\sim\frac{SO(2,1)}{SO(2)}\times\frac{SO(2,1)}{SO(2)}.$$

Finally, we propose the following correspondence given in table 2.

3.2. Dyonic 2-brane attractors

Since the technical analysis of six and eight dimensions are quite similar, we shall only discuss here the dyonic solution[7]. The dyonic black 2-brane is associated with the case of $n=1$ and $p=2$ whose near-horizon geometry is given by $AdS_4\times S^4$. In this configuration, there are no closed 2 and 3-forms, so the corresponding charges are not allowed. There are only 4-form charges (electric and magnetic) supported by this geometry. Using an analysis similar to the one given in [26], the near-horizon geometry ansatz of this configuration can be written as

$$\mathrm{d}s^2=r^2_{AdS}\,\mathrm{d}s^2_{AdS_4}+r^2_S\,\mathrm{d}s^2_{S^4},\qquad G_4=p\,\alpha_{S^4}+e\,\beta_{AdS_4},\tag{3.7}$$

where α_{S^4} and β_{AdS_4} denote respectively the volume forms of S^4 and AdS_4. The magnetic charge p and the electric charge e are defined by

$$p=\int_{S^4}G_4\qquad e=\int_{AdS_4}\star G_4.\tag{3.8}$$

It is useful to introduce the parameterization $Q^{1,2}=\frac{1}{2}(p\pm e)$, so the central charges take the following general form

$$Z=MQ,\tag{3.9}$$

where M is a 2×2 matrix parameterizing the $SO(1,1)$ factor. This matrix is represented by

$$\begin{pmatrix}\cosh(2\phi) & \sinh(2\phi)\\ \sinh(2\phi) & \cosh(2\phi)\end{pmatrix},\tag{3.10}$$

[7] We hope to give a general solution in a future work [28].

where ϕ is the dilaton scalar field. The dyonic black 2-brane effective potential for the dilaton is given as usual in terms of the central charges. Using the equation (3.10), this potential reads as

$$V_{\text{eff}} = \frac{1}{2}(p^2 \exp(-4\phi) + e^2 \exp(4\phi)). \tag{3.11}$$

The dilaton is stabilized at the minimum of the previous potential by the following attractor equations

$$\frac{\mathrm{d}V}{\mathrm{d}\phi} = 0, \qquad \frac{\mathrm{d}^2 V}{\mathrm{d}^2\phi} > 0. \tag{3.12}$$

The solution of these equations is

$$\exp(4\phi) \sim \frac{p}{e}. \tag{3.13}$$

We obtain here exactly the same value of the dilaton that appears in the K3 attractor. In both cases the dilaton can be fixed by the electric and magnetic charges of the dyonic object. The only difference that occur in eight dimensions is that the dilaton can be fixed by black 2-brane, while in six dimensions it has been fixed by black string charges.

4. Conclusions and open questions

We have reconsidered the analysis of the moduli space of $N = 2$ eight-dimensional supergravity. The near-horizon geometries of the corresponding extremal black p-branes have been assumed to be products of AdS_{p+2} with $(6 - p)$-dimensional real spheres S^{6-p}. Inspired from K3-attractors, we have proposed a new three factor realization for the scalar manifold of such a $N = 2$ supergravity model. This form is based on the existence of three different extremal black p-brane solutions with $p = 0, 1, 2$. Using the conjecture introduced in [25], we have identified the black objects associated to each factor. In particular, each coset space has been associated with a eight-dimensional black brane charge solution. The novel feature is that in this case the number of U(1) charges associated to the black objects is larger than the dimension of the scalar factor moduli. However, given the discrete character of the charges, this effect, which is not present in the six-dimensional case, does not imply by any means any redundancy or over counting.

We have also analyzed the attractor mechanism on the elliptic curve, and pointed out a correspondence between the scalar manifold factors and the extremal black object charges. We specially focused on the case of dyonic attractors. In the $AdS_4 \times S^4$ near-horizon limit of the black 2-brane we have shown that upon minimization the effective potential fixes the value of the dilaton in terms of electric and magnetic charges.

An interesting open question concerns the solution of the attractor equations for general extremal black p-branes in eight dimensions. It should be also interesting to look for non supersymmetric attractor solutions on the elliptic curve.

Another open problem is the analysis of extremal black brane attractors on general Riemann surfaces. In particular the study would be interesting for a two-dimensional sphere S^2, where there is only one Kähler parameter controlling its size, and the gauge vectors come only from the **R**–**R** sector. In this case, we guess that the size of S^2 might be fixed by the **R**–**R** black hole charges. We shall address these open questions in the future [28].

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